LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2011

MT 3810 / 3803 - TOPOLOGY

Date : 31-10-2011 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer all questions. All questions carry equal marks. **5 x 20 = 100 marks**

01) (a) (i) Let X be a non-empty set and let d be a real function of ordered pairs of elements of X which satisfies the following conditions.

a) $d(x,y) = 0 \Leftrightarrow x = y$

b) $d(x,y) \leq d(x,y) + d(z,y) \; \forall \; x,y,z \in X$. Show that d is a metric on X.

(or)

(ii) Let X be a metric space. Prove that a subset G of X is open $\Leftrightarrow$ it is a union of open spheres. (5)

(b) (i) Let X be a metric space, and let Y be a subspace of X. Prove that Y is complete iff Y is closed.

(ii) State and prove Cantor's Intersection Theorem.

(iii) State and prove Baire's Theorem. (6+5+4)

(or)

(iv) Let X and Y be metric spaces and let f be a mapping of X into Y. Prove that f is continuous at $x_n \to x_o \Leftrightarrow f(x_n) \to f(x_o)$ and f is continuous $\Leftrightarrow f^{-1}(G)$ is open in X whenever G is open in Y. (15)

02) (a) (i) Prove that every separable metric space is second countable.

(or)

(ii) Define a topology on a non-empty set X with an example. Let X be a topological space and A be an arbitrary subset of X. Show that $\overline{A} = \{x /$ each neighbourhood of x intersects $A\}$. (5)

(b) (i) Show that any continuous image of a compact space is compact.

(ii) Prove that any closed subspace of a compact space is compact.

(iii) Give an example to show that a compact subspace of a compact space need not be closed. (6+6+3)

(or)

(iv) Show that a topological space is compact, if every subbasic open cover has a finite subcover. (15)

03) (a) (i) State and prove Tychnoff's Theorem.

(or)

(ii) Show that a metric space is compact if it is complete and totally bounded. (5)

(b) (i) Prove that in a sequentially compact space, every open cover has a Lebesgue's number.

(ii) Prove that every sequentially compact metric space is totally bounded. (10+5)

(or)

(iii) State and prove Ascoli's Theorem. (15)

04) **(a)** (i) Show that every subspace of Hausdorff space is also Hausdorff.

(or)

(ii) Prove that every compact Hausdorff Space is normal. (5)

(b) (i) Prove that the product of any non-empty class of Hausdorff Spaces is a Hausdorff Space.

(ii) Prove that every compact subspace of a Hausdorff space is closed.

(iii) Show that a one-to-one continuous mapping of a compact space onto a Hausdorff Space is a homeomorphism. (6+4+5)

(or)

(iv) If X is a second countable normal space, prove that there exists a homeomorphism f of X onto a subspace of $\mathbb{R}^{\infty}$ and X is therefore metrizable. (15)

05) **(a)** (i) Prove that any continuous image of a connected space is connected.

(or)

(ii) Show that the components of a totally disconnected space are its points. (5)

(b) (i) Show that the product of any non-empty class of connected spaces is connected.

(ii) Let X be a compact Hausdorff Space. Show that X is totally disconnected, iff it has open base whose sets are also closed. (6+9)

(or)

(iii) State and prove the Weierstrass Approximation Theorem. (15)

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